ON SMALL COHEN-MACAULAY CONJECTURE FOR LOCAL RINGS SUCH THAT DEPTH = DIM−1

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Let A be a Noetherian local ring with maximal ideal $\mathfrak{m}$ and M a finitely generated A-module. Then the following inequalities are well-known:

$$\operatorname{depth} M \leq \dim M \leq \dim A.$$

If $\operatorname{depth} M = \dim M$, then M is called a Cohen-Macalay A-module. If $\operatorname{depth} M = \dim A$, then M is called a maximal Cohen-Macaulay A-module.

Hochster posed the following conjecture:

Small Macaulay module conjecture ([1], p. 10)**.** There is a maximal Cohen-Macaulay A-module if A is complete.

There are few affirmative answer to the conjecture. However, Shimomoto and Tavanfar recently gave remarkable instance of Noetherian local rings having a maximal Cohen-Macaulay modules.

Theorem 1 (A part of Theorem 3.2 of [2])**.** Assume that A is a homomorphic image of a Gorenstein local ring and quasi-Gorenstein ring. If $\dim A = 3$, $\operatorname{depth} A = 2$ and $H^2_{\mathfrak{m}}(A) = A/\mathfrak{m}$, then A has a maximal Cohen-Macaulay module.

In our talk, we generalize their theorem.

Theorem 2. Assume that A is a homomorphic image of a Gorenstein local ring. If $n = \dim A \geq 3$, $\operatorname{depth} A = n - 1$ and $\operatorname{Ext}^{n-1}_A(A/\mathfrak{m}, A) = A/\mathfrak{m}$, then A has a maximal Cohen-Macaulay module.

References

[1] Melvin Hochster, *Topics in the homological theory of modules over commutative rings*, CBMS Reg. Conf. Ser. in Math., vol. 24, Amer. Math. Soc., Providence, RI, 1975.

[2] Kazuma Shimomoto and Ehsan Tavanfar, *Remarks on the small Cohen-Macaulay conjecture and new instances of maximal Cohen-Macaulay modules*, J. Algebra **634** (2023), 667–697.