TILTING THEORY VIA g-FANS IN REAL GROTHENDIECK GROUPS

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The notion of tilting complexes is basic to study the structure of derived categories of rings. The class of silting complexes complements the class of tilting objects from a point of view of mutation.

For a finite dimensional algebra A over a field, 2-term silting complexes of A give rise to a simplicial complex (called the g-simplicial complex) and a nonsingular fan in the real Grothendieck group of A (called the g-fan). For example, the g-fan of a preprojective algebra is the Coxeter fan, and the g-fan of a Jacobian algebra of a certain quiver with potential is the g-fan of the corresponding cluster algebra.

The g-fan of A is a useful combinatorial invariant which has a lot of information about representation theory of A, and therefore satisfies many nice properties. For example, the h-vector and the Dehn-Sommerville equation of the g-simplicial complex has a tilting theoretic interpretation. Also the g-fan of A is complete if and only if A has only finitely many 2-term silting complexes up to isomorphism.

One of the basic problems is to classify complete g-fans. We give an answer for rank 2 case by showing that complete g-fans of rank 2 are precisely complete sign-coherent fans of rank 2. As a by-product, for each positive integer N , we gives a finite dimensional algebra A of rank 2 such that the Hasse quiver of the poset of 2-term silting complexes of A has precisely N connected components.

This talk is based on a series of joint works with T. Aoki, A. Higashitani, R. Kase and Y. Mizuno [AHIKM1, AHIKM2].

References

[AHIKM1] T. Aoki, A. Higashitani, O. Iyama, R. Kase, Y. Mizuno, Fans and polytopes in tilting theory I: Foundations, arXiv:2203.15213

[AHIKM2] T. Aoki, A. Higashitani, O. Iyama, R. Kase, Y. Mizuno, Fans and polytopes in tilting theory II: g-fans of rank 2

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