SEMINORMALITY AND GRADED RINGS

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This talk is based on [1]. The complete version of this research will be submitted to elsewhere.

1. Introduction

Let A be a commutative Noetherian ring. $I \subseteq A$ is a proper ideal. For any integers $n \le 0$, we set $I^n = A$. With these notation, we define the *Rees ring of* I as $\mathscr{R}_+(I) := \bigoplus_{n \ge 0} I^n t^n$, *extended Rees ring of* I as $\mathscr{R}(I) := \bigoplus_{n \in \mathbb{Z}} I^n t^n$, and the *associated graded ring of* I as $G(I) := \mathscr{R}_+(I)/I\mathscr{R}_+(I) = \bigoplus_{n \ge 0} I^n/I^{n+1}$ respectively. The purpose of this talk is to present the following result. For the definitions of seminormal ring and weak normal ring, we recommend survey paper [2] for a reference.

Theorem 1.1. *$(A, \mathfrak{m})$ is a Noetherian local ring of $\dim A \ge 1$. I is an ideal of A. Then we have the followings.*

(1) If $G(I)$ is seminormal, then $\mathscr{R}_+(I)$, $\mathscr{R}(I)$ and A are seminormal.
(2) If $G(I)$ is weakly normal, then $\mathscr{R}_+(I)$, $\mathscr{R}(I)$ and A are weakly normal.

Let us remind that similar result hold for the normality. To prove above result, we need the "Matijevic-Roberts-type" theorem for seminomality and weak normality.

Theorem 1.2. *$A = \bigoplus_{n \in \mathbb{Z}} A_n$ is a graded Noetherian ring. Suppose that the integral closure of A in its total ring of fractions is $\mathbb{Z}$-graded. Then we have the followings.*

(1) A is seminormal if and only if its localization $A_\mathfrak{m}$ is seminormal, where $\mathfrak{m}$ is any graded maximal ideal of A.
(2) A is weakly normal if and only if its localization $A_\mathfrak{m}$ is weakly normal, where $\mathfrak{m}$ is any graded maximal ideal of A.

References

[1] J. Horiuchi and K. Shimomoto, *Matijevic-Roberts type theorems, Rees rings and associated graded rings*, in preparation.
[2] M. Vitulli, *Weak normality and seminormality*, Commutative algebra: Noetherian and non-Noetherian perspectives, Springer-Verlag, (2011), 441–480.

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